NUMERICAL STUDY OF THE TWO-DIMENSIONAL FLOW OF A BINARY MIXTURE WITH A VAPOR–ICE PHASE TRANSITION AT THE CHANNEL WALL

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An analysis is made of the problem of the desublimation of vapor from a laminar stream of a vapor–air mixture in a plane channel with allowance for the changes in the boundary conditions connected with the buildup of a layer of ice.

The process of desublimation of vapor in vacuum condensers has a number of specific features distinguishing it from the ordinary process of condensation into the liquid state. This is primarily connected with the variation in the conditions at the surface in the course of the process owing to the buildup of the desublimate. The existing methods of calculating the process of desublimation allow for only individual aspects of the phenomenon [1-3], and their application is limited.

The problem of the desublimation of vapor from a vapor–air mixture moving in a channel, one wall of which has a temperature T_c below the dew-point temperature while the temperature T_h of the other is above the dew-point temperature, which is important in a practical respect, is analyzed here.

The parameters of the process, such as the temperatures of the plates and the inlet temperature of the stream and the flow rate and pressure of the mixture, are kept constant. Taking the process as quasisteady and the thermophysical properties of the mixture as constants, we write the equations of vorticity and of the conservation of mass, momentum, and energy in the form [4]

$$\frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial\psi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial\psi}{\partial y}\right)+\omega=0, \tag{1}$$

$$\frac{\partial}{\partial x}\left(\omega\frac{\partial\psi}{\partial y}\right)-\frac{\partial}{\partial y}\left(\omega\frac{\partial\psi}{\partial x}\right)=\frac{1}{\mathrm{Re}}\Delta\omega, \tag{2}$$

$$\frac{\partial}{\partial x}\left(\theta\frac{\partial\psi}{\partial y}\right)-\frac{\partial}{\partial y}\left(\theta\frac{\partial\psi}{\partial x}\right)=\frac{1}{\mathrm{RePr}}\Delta\theta, \tag{3}$$

$$\frac{\partial}{\partial x}\left(m\frac{\partial\psi}{\partial y}\right)-\frac{\partial}{\partial y}\left(m\frac{\partial\psi}{\partial x}\right)=\frac{1}{\mathrm{ReSc}}\Delta m. \tag{4}$$

We neglect the variation in the total pressure in the field of flow, so that the density ρ of the mixture is found from the equation

$$\rho=A(m+B)^{-1}(\theta+C)^{-1}, \tag{5}$$

where

$$A=(m_0+B)(1+C),\ B=M_1/(M_2-M_1),\ C=T_c/(T_0-T_c).$$

A diagram of the flow is shown in Fig. 1.

The boundary conditions at the cooled plate for a vortex intensity ω can be obtained from (1) and (2) under the assumption that the gradients in the direction parallel to the wall are considerably less than the gradients normal to it. We obtain the connection between the stream function and the vorticity in the form [4]

$$\psi(y)-\psi_c=-\int_0^y\rho(\eta)\left[\int_0^\eta\omega d\xi\right]d\eta, \tag{6}$$

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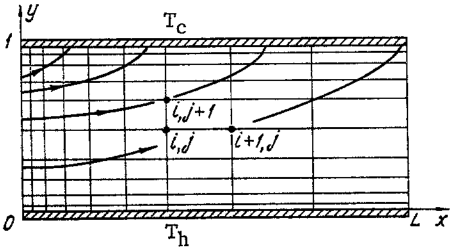

Fig. 1

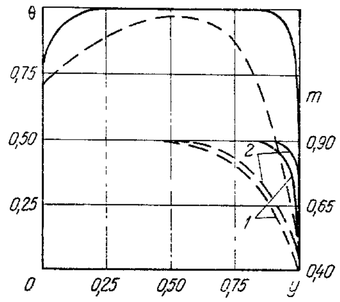

Fig. 2

Fig. 1. Diagram of the region of flow with the finite-difference grid.

Fig. 2. Profiles of the temperature θ and mass concentration m in different channel cross sections: solid curves) x = 11.71; dashed curves) 0.33; 1) at $\tau = 0$; 2) $3 \cdot 10^3$.

$$\omega(y) = \left(A_1 \int_0^y f(\eta)\, d\eta + B_1\right)/f(y), \tag{7}$$

where A_1 and B_1 are integration constants; $f(y) = \exp\left[\mathrm{Re} \int_0^y \left(\frac{\partial\psi}{\partial x}\right) d\xi\right]$.

The derivative $\partial\psi/\partial x$ entering into (7) has the meaning of the flux of matter to the wall. This flux is due to the condensation of only one component of the mixture – the water vapor. This phenomenon is accompanied by convective motion of the mixture – Stefan flow [5, 6]:

$$I(x, \tau) = \frac{\partial\psi}{\partial x} = \frac{1}{\mathrm{ReSc}} \frac{\partial \ln(1-m)}{\partial y}. \tag{8}$$

Substituting (8), which in turn is the boundary condition at the desublimation surface for the stream function ψ, into (7) and performing the integration in the boundary region, we obtain the boundary condition for the vortex intensity at the absorbing surface:

$$\omega_c = -\left[\frac{3(\psi(y) - \psi_c)}{\rho y^2} + \frac{\omega(y)}{2}\left(\frac{7+D}{8}\right)\right] E^{-1}, \tag{9}$$

where $D = [(1 - m(y))/(1 - m_i)]^{1/\mathrm{Sc}}$ and $E = [11 + 5D - (7 + D)\ln D]/16$. The temperature at the desublimation surface is determined from the conjugation condition at the vapor – ice boundary [7, 8]:

$$q_{he} + q_{con} = q, \tag{10}$$

$$\theta_v = \theta_i. \tag{11}$$

where q_{he} is the flux due to molecular heat conduction; q_{con} is the heat flux due to vapor condensation onto the surface.

The assumption that the process is quasisteady allows one to adopt a linear temperature distribution in the layer of desublimate and to write the condition (10), with allowance for (8), in the form

$$\left(\frac{\partial\theta}{\partial y}\right)_v + \frac{\mathrm{Pr}}{\mathrm{ScSte}} \cdot \frac{1}{1-m} \frac{\partial m}{\partial y} = \frac{\lambda_i}{\lambda_v} \frac{(\theta_c - \theta_i)}{h(x, \tau)}. \tag{12}$$

The rate of movement of the ice surface is

$$\frac{\partial h}{\partial \tau} = \frac{I(x, \tau)}{\rho_i}. \tag{13}$$

The conditions (11)-(13) determine the law of motion of the phase interface and the temperature θ_i at this boundary. Neglecting the temperature and concentration jumps [9, 10] at the desublimation surface, we write the boundary condition for the mass concentration in the form

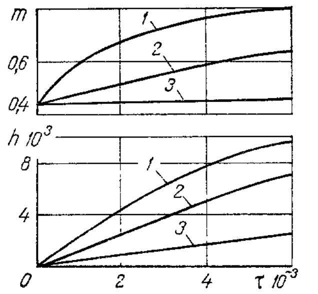

Fig. 3

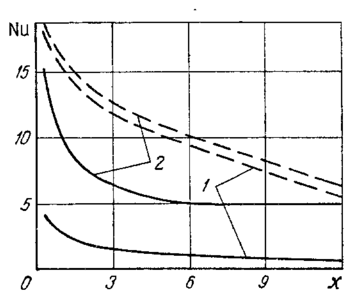

Fig. 4

Fig. 3. Time dependences of mass concentration at the ice surface and of thickness of the ice layer: 1) $x = 0.33$; 2) 2.04; 3) 11.71.

Fig. 4. Distribution of the local Nusselt number along the walls: solid curves) at hot wall; dashed curves) at cold wall; 1, 2) based on Eqs. (18) and (19), respectively.

$$m_i = \left[\frac{M_2}{M_1}\frac{1}{P_{sat}(\theta_i)} + \left(1 - \frac{M_2}{M_1}\right)\right]^{-1}, \tag{14}$$

where $P_{sat}(\theta_i)$ is the saturation pressure.

Thus, Eqs. (8), (9), and (11)-(14) give the boundary conditions at the surface of the desublimate.

Since the hotter plate is impermeable, the boundary conditions at it can be written in the form

$$\omega_h = -\frac{3(\psi(y) - \psi_h)}{\rho y^2} - \frac{\omega(y)}{2},$$
$$\psi_h = 0, \quad \theta = \theta_h, \quad \partial m/\partial y = 0. \tag{15}$$

The equation for the vorticity ω is obtained from (9) with $D = 1$ (the absence of mass exchange), which agrees with the results of [4].

At the channel inlet the stream is undisturbed, so that the conditions at the boundary have the form

$$\omega = 0, \quad \psi = y, \quad \theta = 1, \quad m = m_0. \tag{16}$$

The conditions in the outlet cross section required for closure of the problem are not known a priori. Assuming that the flow is stabilized and the channel length is great enough that the influence of the outlet on the initial section is small, we write the conditions at the outlet:

$$\frac{\partial\omega}{\partial x} = \frac{\partial\psi}{\partial x} = \frac{\partial\theta}{\partial x} = \frac{\partial m}{\partial x} = 0. \tag{17}$$

It should be noted that the conditions (8)-(14) occurring at the ice surface can be referred to the surface of the plate, since the thickness of the desublimate is assumed to be sufficiently small in comparison with the distance between the plates. This allows one to assume that the normal to the ice surface coincides with the y direction on the entire plate except for the initial section, where one must allow for the horizontal components of the heat and mass fluxes.

To solve the stated problem we used the difference method presented in [4]. A diagram of the flow with the finite-difference grid is presented in Fig. 1. The greatest density of grid lines falls in the regions with the largest gradients of the dependent variables: the inlet cross section and the boundary layers at both plates. The integration of Eq. (13) was carried out with a variable time step, with correction of the step as a function of the size of the mass flux to the wall. The finite-difference equations were solved by the Gauss — Zaidel' method of successive displacements. The criterion for convergence of the iterations with respect to the variables ψ, θ, and m is $|1 - f_i/f_{i-1}| < \varepsilon$, while for the intensity of the vorticity ω it is $|(\omega_{i-1} - \omega_i)/\omega_{i-1}^{max}| < \varepsilon_1$, where ε and ε_1 were taken as 0.01 and 0.0001, respectively; i is the iteration number; the index max denotes the maximum absolute value of ω in the entire flow field. As the initial approximation for the procedure of successive displacements in the next time step we took the values at the end of the preceding step.

The numerical solution makes it possible to obtain a rather complete picture of the development of the process of heat and mass exchange in a wide range of variation of the technological parameters.

The profiles of temperature and concentration at x = 0.33 and x = 11.71 with a total channel length L = 100 for $\tau = 0$ and $\tau = 3 \cdot 10^3$ for a vapor — air mixture with an initial vapor concentration $m_0 = 0.9$ in the stream (Re = $1.1 \cdot 10^3$, $T_0 = 323°K$, $T_h = 293°K$, $T_c = 238°K$, $P_0 = 0.3$ mm Hg) are shown in Fig. 2 as an example. As one would expect, the temperature profile is asymmetric and its maximum is shifted toward the cold surface: the suction effect is manifested. Nevertheless, in an analysis of the concentration profile it is seen that the bulk of the vapor has not condensed in a section of length x = 11.71. For these conditions the efficiency of the desublimation surface is evidently inadequate, and one must intensify the process of mass exchange to assure fuller freezing out of the stream.

The results on the time variation of the vapor concentration at the surface of the desublimate (Fig. 3) and of the thickness of the ice layer in different channel cross sections are of independent interest. The behavior of these characteristics indicates that in the inlet section the process of desublimation of vapor from the vapor — gas mixture is limited mainly by the thermal resistance of the layer of desublimate, since the diffusional boundary layer is thin. In later cross sections the growth of the desublimate is determined by the rate of vapor transport to the cold surface, i.e., by the diffusional resistance.

The distribution of the local values of Nu (Fig. 4) is of definite interest from the point of view of practical engineering. At the desublimation surface (as at the hotter wall) the values of Nu were calculated from two equations:

$$\mathrm{Nu} = \left| \frac{\partial \theta}{\partial y} \right|, \tag{18}$$

$$\mathrm{Nu} = \frac{1}{\langle \theta \rangle} \left| \frac{\partial \theta}{\partial y} \right|, \tag{19}$$

where $\langle \theta \rangle$ is the calorimetric mean value of the temperature over a channel cross section [12]:

$$\langle \theta \rangle = \int_0^{\psi_i} \frac{\theta(y)}{\rho(y)}\, d\psi(y) / \int_0^{\psi_i} \frac{d\psi(y)}{\rho(y)} - \theta_i. \tag{20}$$

As is seen, the value of Nu is quite variable at the desublimation surface. Therefore, calculating methods based on constancy of the coefficient of heat transfer at the desublimation surface should be used with some caution.

NOTATION

H, distance between plates, scale of length, m; (x, y), L, h, dimensionless coordinates, length of plates, and thickness of ice layer, respectively; V, velocity of gas mixture, m/sec; T, temperature, °K; ψ, ω, θ, m, ρ, q, I, P, τ, stream function, vorticity, temperature, mass concentration, density, heat flux, mass flux, pressure, and time, respectively, all dimensionless; $Sc = \mu/(\rho D')$, Schmidt number; $Pr = \mu c_p/\lambda$, Prandtl number; $Re = \rho_0 V_0 H/\mu$, Reynolds number; $Ste = c_p(T_0 - T_c)/r$, Stefan number; $Nu = \alpha H/\lambda$, Nusselt number; i, j, indices of finite-difference system; Δ, Laplace operator; M_1, M_2, molecular weights of water and air, respectively. Indices: 0, inlet cross section; c, cold plate; h, hot plate; v, vapor — air mixture; i, ice surface.

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REFINED CALCULATION OF SUPERSATURATION DURING HEAT AND MASS EXCHANGE IN A STATIONARY GAS MEDIUM

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Equations are presented for calculating the supersaturation in the absence of convection with allowance for the Stefan flow.

The appearance of a fog during the condensation of a vapor is observed in the most varied industrial and natural phenomena. In the production of sulfuric, phosphoric, and other acids, e.g., a stable and corrosive fog forms, a considerable part of which passes through the filters and is discharged into the atmosphere. The obtainment of fine powders of metals by the distillation method is accompanied by volume condensation. Processes of fog formation are also used for purposes of scientific research.

In the indicated cases the jointly occurring processes of heat and mass exchange are accompanied by a rise in the vapor pressure to values exceeding the saturation vapor pressure at the given temperature above a plane surface. The ratio $S = p/p(T)$ is called the supersaturation. But the supersaturation is limited to its critical value for each case. The value of the critical supersaturation depends both on the presence of suspended particles and on the presence of gas ions, and it can differ markedly from unity in a sufficiently purified medium [1].

When the critical supersaturation is reached the process changes qualitatively — the formation of a fog begins — and therefore in an analysis of a problem concerning volume condensation one must know whether the supersaturation has reached the critical value.

Let us consider the problem of determining the supersaturation profile in a gap formed by wet porous or solid wetted plane-parallel surfaces. To reduce to a minimum the phenomena connected with the occurrence of natural convection, we place the evaporator on top and the condenser on the bottom (Fig. 1). The gap is filled with a gas. The dimensions of the plates in the horizontal directions are so great that the problem can be treated as one-dimensional.

A simple solution to the problem under consideration was obtained in [1] on the basis of the equations $D(d^2P_1/dx^2) = 0$ and $a(d^2T/dx^2) = 0$, the integrals of which have the form

$$P_1 = P - \frac{P_e - P_c}{\delta} x, \tag{1}$$

$$T = T_e - \frac{T_e - T_c}{\delta} x. \tag{2}$$

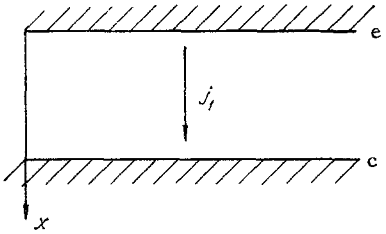

Fig. 1. Schematic representation of the process.

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